On the Optimal Pricing and Subvention Policies in Multimodal Trip Chains

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Abstract

This paper presents a model addressing optimal pricing and subvention policies of multimodal trip chains consisting of the feeder service and trunk line. First best and second best optimal pricing are studied in a situation where travelers can choose between private car trip and multimodal trip chain by public transport. The model is illustrated with numerical examples and the results are compared to the current pricing and subvention policies in Finland.

1 Introduction

Multimodal transport pricing and subvention policies have been studied extensively during last decades (Small and Verhoef, 2007; Ahn, 2009; Tirachini and Hensher, 2012; Tirachini et al., 2014). Tirachini and Hensher (2012) provides review on previous research considering multimodal pricing and various related aspects such as the optimal frequency and capacity of public transport, crowding in public transport, dedicated bus lines, congestion interaction between transport modes and the other externalities of transportation. Moreover, they provide a model including non-motorized transport for an analysis of optimal multimodal pricing.

Multimodal trip chains, i.e., trips travelled by using different transport modes for different parts of the trip are common for public transport users in urban areas where various and complementary transport modes are provided. For example, a trip from home to the railway station by bus and then by train to the final destination is a typical multimodal trip chain in Finland and elsewhere in Europe. Seamless multimodal trip chains with certain connections and minimal transfer times are seen as an essential area of development for improving attractiveness of public transportation, which is recently also mentioned in the national transport sector growth program in Finland as one of the measures for improving competitiveness of cities (TEM, 2017).

In this paper, we analyze optimal pricing (first and second best) and subvention of multimodal trip chains with the model where travelers have two alternatives, private car trip (from door to door) and multimodal public transport, which consists of a feeder service (taxi or minibus) and a train service. Congestion externalities are assumed in private car traffic, but there is no congestion interaction between modes, i.e., the feeder service is assumed to be operated on uncongested areas or on dedicated lines. Moreover, we consider also other transportation externalities than congestion (such as pollution and noise) as in the model presented by Tirachini and Hensher (2012).

In general, optimal pricing of public transportation and arguments for subsidies (economies of scale and private car congestion) are well known (Small and Verhoef, 2007), but pricing and subvention of complementary feeder services is less studied, and the goal of the paper is to provide an analysis and understanding on this issue, which is relevant for transport policies aiming for improving attractiveness of multimodal trip chains in public transport services together with complementary feeder services enabled by technological advancements and related service innovations such as autonomous minibuses and the Mobility as a Service (MaaS) concept.

Finally, we present some numerical illustrations of the model and compare the results to the prevailing pricing policies in Finland where public transportation is currently mostly subsidized by local authorities (municipalities). Separated subvention decisions can be challenging from the viewpoint of the optimal subvention of multimodal trip chains reaching over several municipalities as the benefits and costs of transportation services are not usually distributed evenly between municipalities.

2 Model of multimodal trip chain

We present a model where travelers have two alternatives for trip from A to C, private car trip (from door to door, i.e., A to C) and multimodal public transport trip chain, which consists of a feeder service (taxi or minibus) from A to B and a train service from B to C. The both trip alternatives use similar routes from A to B, and then from B to C, i.e., describing a common situation where a trip from suburban or rural area to the train station and alternatively to the highway junction are similar as well as the routes from the station and the highway junction to the city center (B to C). Of course, the routes are not exactly the same, but the trip distances are almost the same in these trip alternatives. Thus, for simplicity, we assume that distances D_{AB} and D_{BC} are the same for both transport mode alternatives.

We use similar notation than in the model of Tirachini and Hensher (2012) which also follows the notation of Small and Verhoef (2007). The demand for trips from A to C by alternative transport modes, i.e, a private car and a multimodal public transport can be obtained from the benefit function $B(q_c, q_p)$ defining consumers' willingness to pay for a combination $\{q_c, q_p\}$, where q_c is a quantity of private car trips and q_p is a quantity of multimodal public transport trips. The inverse demand function is defined by:

$$d_i(q_c, q_p) = \frac{\boldsymbol{B}(q_c, q_p)}{\partial q_i} \quad i \in \{c, p\} \tag{1}$$

Total cost of mode i, C_i is defined by:

$$C_i = q_i c_i (D_{AB} + D_{BC}) \tag{2}$$

where c_i is an average cost of mode i (c or p) per kilometer. There are congestion costs in the car trips and crowding costs in the public transport, but there is no congestion interaction between modes. Thus, the average cost of car trip is just a function of quantity of car trips, $c_c(q_c)$ and the average cost of trip by feeder service is a function of quantity of passengers, $c_f(q_p)$. The average cost of train (trunk line) trip is a function of quantity of passengers, service frequency (f_t), and vehicle capacity (K_t), $c_t(q_p, f_t, K_t)$. Respectively, the average cost of multimodal trip chain is a sum of the average costs of the feeder service and train service:

$$c_p = c_f + c_t \tag{3}$$

This includes both the user costs and the operator costs, and can be also presented separately:

$$c_p = c_u + c_o = c_{u,f} + c_{o,f} + c_{u,t} + c_{o,t} \tag{4}$$

Thus, the total cost of multimodal public transport is defined by:

$$C_p = q_p c_f (D_{AB}) + q_p c_t (D_{BC}) \tag{5}$$

In equilibrium, marginal benefits are equal to generalized prices of trip alternatives:

$$\frac{\partial B}{\partial q_c} = c_c (D_{AB} + D_{BC}) + \tau_c \tag{6}$$

$$\frac{\partial B}{\partial q_p} = c_{u,f}(D_{AB}) + c_{u,t}(D_{BC}) + \tau_p \tag{7}$$

where τ_c is the road use charge, τ_p is the price of multimodal trip chain (by public transport).

The social planner's objective is to maximize welfare defined as the benefits minus costs with regard to q_c , q_p, f, and K, subject to the capacity constraint of public transport trunk line.

$$SW = B\big(q_c, q_p\big) - q_c\, c_c(q_c)(D_{AB} + D_{BC}) - q_p\, c_t\big(q_p, f_t, K_t\big)(D_{BC}) - q_p c_f\big(q_p\big)(D_{AB})$$

$$-v_c\, q_c EC_c(q_c)(D_{AB} + D_{BC}) - f_t EC_t\big(q_p, f_t, K_t\big)(D_{BC}) - v_f\, q_p EC_f\big(q_p\big)(D_{AB}) \tag{8}$$

$$f_t K_t \le q_p \tag{9}$$

where $EC_c(q_c)$ is the average external cost of car per kilometer, $EC_t\big(q_p, f_t, K_t\big)$ is the average external cost of trunk line per kilometer, $EC_f\big(q_p\big)$ is the average external cost of feeder service per kilometer, $v_c$is the inverse of the average occupancy rate of car and v_f is the same for feeder service.

2.1 First best pricing

The Lagrangean function for the maximization problem can be formulated as:

$$L = B\big(q_c, q_p\big) - q_c\, c_c(q_c)(D_{AB} + D_{BC}) - q_p\, c_t\big(q_p, f_t, K_t\big)(D_{BC}) - q_p\, c_f\big(q_p\big)(D_{AB}) -$$

$$v_c\, q_c EC_c(q_c)(D_{AB} + D_{BC}) - f_t EC_t\big(q_p, f_t, K_t\big)(D_{BC}) - v_f\, q_p EC_f\big(q_p\big)(D_{AB}) + \lambda_1\big[f_t K_t - q_p\big]$$

(10)

where λ_1 is the Lagrange multiplier related to the capacity constraint of the public transport trunk line. This model enables analysis of optimal frequency (f_t), vehicle capacity (K_t), and road use pricing. However, in the following we first focus on the analysis of public transport pricing in the case where capacity constraint is not binding, i.e., in this case $\lambda_1 = 0$. For defining optimal pricing of multimodal public transport we derive the first order condition related to q_p:

$$\frac{\partial L}{\partial q_p} = \frac{\partial B}{\partial q_p} - c_t(D_{BC}) - q_p \frac{\partial c_t}{\partial q_p}(D_{BC}) - f_t \frac{\partial EC_t}{\partial q_p}(D_{BC}) - c_f(D_{AB}) - q_p \frac{\partial c_f}{\partial q_p}(D_{AB}) - v_f EC_f(D_{AB}) -$$

$$v_f q_p \frac{\partial EC_f}{\partial q_p}(D_{AB}) = 0 \tag{11}$$

By substituting the equilibrium condition (7) to the first order condition and after rearranging the equation we get the optimal first best pricing of the multimodal trip for public transport:

$$\tau_p = c_{o,t}(D_{BC}) + c_{o,f}(D_{AB}) + q_p \frac{\partial c_t}{\partial q_p}(D_{BC}) + f_t \frac{\partial EC_t}{\partial q_p}(D_{BC}) + q_p \frac{\partial c_f}{\partial q_p}(D_{AB}) + v_f\, EC_f(D_{AB}) + v_f q_p \frac{\partial EC_f}{\partial q_p}(D_{AB}) \equiv \tau_p^{FB} \quad (12)$$

2.2 Second best pricing

In the second best pricing we assume that there is no road pricing for private cars, i.e., $\tau_c = 0$. As previously, we analyze the case where capacity constraint of public transport is not binding, i.e., $\lambda_1 = 0$, and the capacity constraint disappears. The Lagrangean function for the maximization problem can be formulated as:

$$L = B(q_c, q_p) - q_c\, c_c(q_c)(D_{AB} + D_{BC}) - q_p\, c_t(q_p, f_t, K_t)(D_{BC}) - q_p\, c_f(q_p)(D_{AB})$$
$$- v_c\, q_c EC_c(q_c)(D_{AB} + D_{BC}) - f_t EC_c(q_p, f_t, K_t)(D_{BC}) - v_f\, q_p EC_f(q_p)(D_{AB})$$
$$+\lambda_2 \left[c_c(D_{AB} + D_{BC}) - \frac{\partial B}{\partial q_c}\right] + \lambda_3 \left[c_{u,f}(D_{AB}) + c_{u,t}(D_{BC}) + \tau_p - \frac{\partial B}{\partial q_p}\right] \quad (13)$$

For defining optimal pricing of multimodal public transport we derive the first order conditions related to q_c, q_p and τ_p:

$$\frac{\partial L}{\partial q_p} = \frac{\partial B}{\partial q_p} - c_t(D_{BC}) - q_p \frac{\partial c_t}{\partial q_p}(D_{BC}) - f_t \frac{\partial EC_t}{\partial q_p}(D_{BC}) - c_f(D_{AB}) - q_p \frac{\partial c_f}{\partial q_p}(D_{AB}) - v_f\, EC_f(D_{AB}) - v_f q_p \frac{\partial EC_f}{\partial q_p}(D_{AB}) + \lambda_2 \left[-\frac{\partial^2 B}{\partial q_p q_c}\right] + \lambda_3 \left[\frac{\partial c_{u,f}}{\partial q_p}(D_{AB}) + \frac{\partial c_{u,t}}{\partial q_p}(D_{BC}) - \frac{\partial^2 B}{\partial^2 q_p}\right] = 0 \quad (14)$$

$$\frac{\partial L}{\partial q_c} = \frac{\partial B}{\partial q_c} - c_c(D_{AB} + D_{BC}) - q_c \frac{\partial c_c}{\partial q_c}(D_{AB}+D_{BC}) - v_c\, EC_c(D_{AB}+D_{BC}) - v_c q_c \frac{\partial EC_c}{\partial q_c}(D_{AB}+D_{BC}) + \lambda_2 \left[\frac{\partial c_c}{\partial q_c}(D_{AB} + D_{BC}) - \frac{\partial^2 B}{\partial^2 q_c}\right] + \lambda_3 \left[-\frac{\partial^2 B}{\partial q_c q_p}\right] = 0 \quad (15)$$

$$\frac{\partial L}{\partial \tau_p} = \lambda_3 = 0 \quad (16)$$

Thus, we get $\lambda_3 = 0$. By substituting this value to the first order condition ($\frac{\partial L}{\partial q_c} = 0$) we get expression for λ_2, which we substitute with the equilibrium condition (7) to the firs order condition ($\frac{\partial L}{\partial q_p} = 0$) to derive the optimal (second best) pricing of the multimodal trip for public transport:

$$\tau_p^{SB} = \tau_p^{FB} - \left(q_c \frac{\partial c_c}{\partial q_c}(D_{AB}+D_{BC}) + v_c\, EC_c(D_{AB}+D_{BC}) + v_c q_c \frac{\partial EC_c}{\partial q_c}(D_{AB}+D_{BC})\right) \frac{-\frac{\partial^2 B}{\partial q_p q_c}}{\frac{\partial c_c}{\partial q_c}(D_{AB}+D_{BC}) - \frac{\partial^2 B}{\partial^2 q_c}}$$

(17)

The second best price, τ_p^{SB}, equals the non-internalized marginal cost of public transport (τ_p^{FB}) less a term that multiplies the marginal congestion externality and other external costs of private car $\left(q_c \frac{\partial c_c}{\partial q_c}(D_{AB}+D_{BC}) + v_c\, EC_c(\, D_{AB}+D_{BC}) + v_c q_c \frac{\partial EC_c}{\partial q_c}(\, D_{AB}+D_{BC})\right)$ by a weight depending on the slope of the inverse demand curve of private car and the cross-effect $\left(-\frac{\partial^2 B}{\partial q_p q_c}\right)$. The weight equals the number of new private car trips per trips deterred from public transport. The second best pricing (17) can be interpreted similarly as the pricing derived by Small and Verhoef (2007) and Tirachini and Hensher (2012), but now other externalities than congestion are also included and distances travelled by different transport modes are presented separately.

3 Numerical examples

For numerical illustration of the model, we consider example cases where passengers travel from suburban (short distances) or rural areas (longer distances) to the center of Helsinki. We use the parameter values presented in Table 1. The operation cost of trunk line per passenger kilometer is based on the reported train costs of the Helsinki Region Transport in 2016 (HSL, 2017). The operation cost of feeder service is based on taxi prices in Finland in 2015 (ELY, 2014), which can be interpreted as the maximum price of the feeder service paid by the public transport operator producing multimodal trip chains. Marginal cost values for private car and trunk line (train) are based on estimations of Parry and Small (2009) (the measurement units are changed), where the negative values of trunk line are based on the estimated scale economies of train services, which are higher than crowding and other cost components especially during off-peak periods. The values for the marginal costs of the feeder service are based on our assumption that private taxis are used for feeder in the trip chains (other options could be, for instance, shared taxis or demand responsive transportation services). The average external cost of feeder service and car are based on estimations of Litman (2016).

Table 1
Values of parameters based on the literature and assumptions on the considered example cases

Parameters / Variables	Values
Operation cost of trunk line per passenger km, $c_{o,t}$	0.11
Operation cost of feeder service per passenger km, $c_{o,f}$	2.73
Marginal cost of trunk line per passenger km, $q_p \frac{\partial c_t}{\partial q_p}$ (peak)	-0.017
Marginal cost of trunk line per passenger km, $q_p \frac{\partial c_t}{\partial q_p}$ (off-peak)	-0.084
Marginal cost of feeder service per passenger km, $q_p \frac{\partial c_f}{\partial q_p}$ (peak)	0
Marginal cost of feeder service per passenger km, $q_p \frac{\partial c_f}{\partial q_p}$ (off-peak)	-0.084
Marginal congestion cost of car per km, $q_c \frac{\partial c_c}{\partial q_c}$ (peak)	0.109
Marginal congestion cost of car per km, $q_c \frac{\partial c_c}{\partial q_c}$ (off-peak)	0.010
Marginal external cost of trunk line per passenger km, $\frac{\partial EC_t}{\partial q_p}$	0
Marginal external cost of feeder service per passenger km, $q_p \frac{\partial EC_f}{\partial q_p}$	0
Average external cost of feeder service per passenger km, EC_f	0.086
Marginal external cost of car per passenger km, $q_c \frac{\partial EC_c}{\partial q_c}$	0.036
Average external cost of car per passenger km, EC_c	0.086
v_c, v_f	1

We calculated numerical values for the first best and second best prices for three trip chains with variable trunk line travel distances (10, 20 and 40 kilometers) and feeder taxi trip of 5 kilometers. Thus, variable total distances were 15, 25 and 45 kilometers. We don't have empirical values for the weight of equation (17), which equals the number of new private car trips per trips deterred from multimodal public transport. Therefore, we varied the value of the weight from 0.1 to 0.9 for illustrating how the optimal prices change with alternative weights describing different market conditions. The prices are presented in Figure 1, where the first best price for peak period is higher than second best prices for peak period as expected as the private car congestion costs are internalized by the road pricing. The first best price for peak period is also higher than for off-peak period due to the higher scale economies of public transport during off-peak period, whereas the second best prices are higher for off-peak period if the weight is 0.5 or higher, because the marginal congestion costs of cars decrease the second best price significantly during peak periods. The higher weights in the second best pricing naturally decrease prices. The weight value 0.9 is probably usually unrealistic high (expect when adopted for special customer segments with inelastic trip demand), but it helps to illustrate the significance of the weight in the second best pricing.

The first best prices increase due to the longer distances by the trunk line, whereas the second best prices decrease with longer distances if the weight is 0.5 or higher, because the production costs of the train per passenger kilometer are relatively low compared to congestion costs and other external costs of cars. We calculated the prices also in the case were the train in the trunk line was replaced by bus with higher costs of 0.236 euros per kilometer (HSL, 2017). In this case the all prices were increasing due to longer distances.

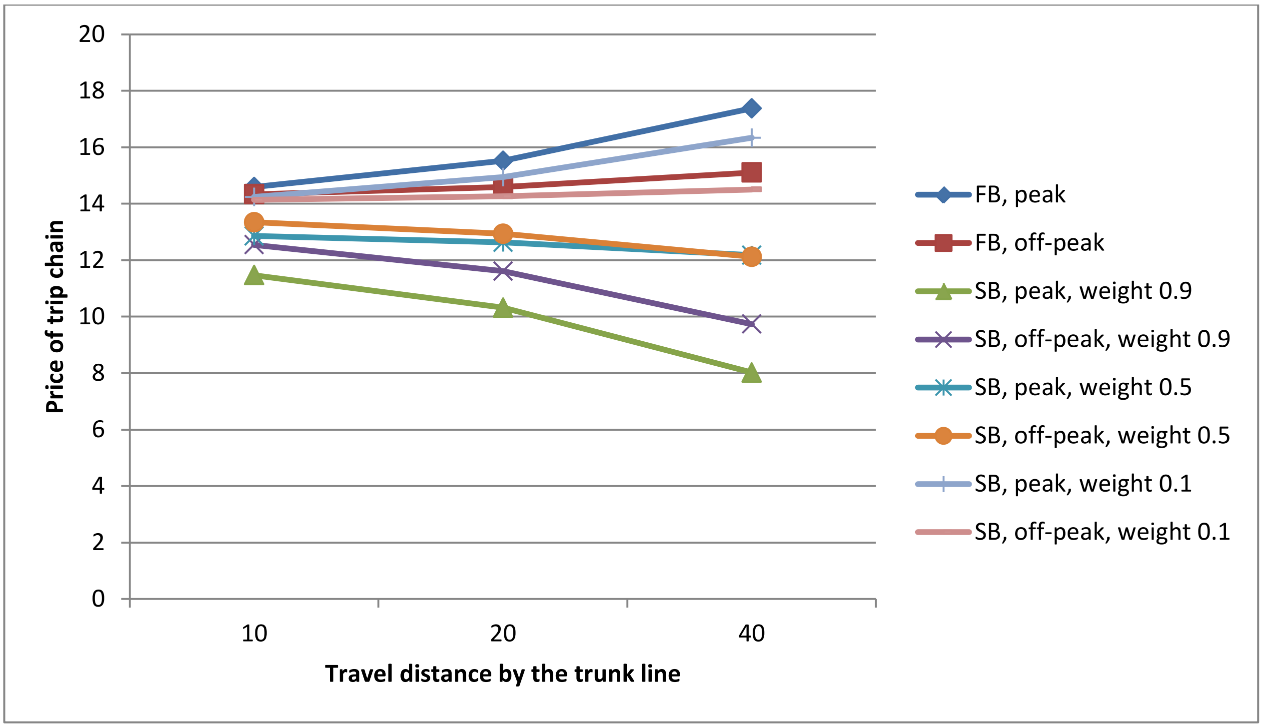

Figure 1. First best and second best prices of the multimodal trip chain (taxi feeder and trunk line) with various market conditions.

We also compared the first best and second best prices with the actual prices of multimodal trip chains to the center of the Helsinki in prices of year 2015 (i.e., taxi and train prices). Moreover, we compared the current prices of the Whim service, which sells alternative bundles of transportation services including public transportation season tickets and short taxi trips (maximum trip distance is 5 kilometers) with lower prices. The prices are presented in Figure 2. The prices of the public transport in 2015 are clearly higher than the first best price, but these prices are the maximum prices as these are based on the prices of single tickets, whereas majority of sold tickets are cheaper (season tickets and discounts for students and other special groups) and the average revenue per trip is approximately only 0.90 euros. Moreover, taxi trips are subsidized only for special groups. The price of the Whim service is based on the bundle (Whim Urban), which includes a season ticket of Helsinki Region Transport and taxi trips by 10 euros. Interestingly, the Whim price is close to the second best price (with the weight 0.5), but increases with the distances due to the more expensive season ticket, which covers wider service area.

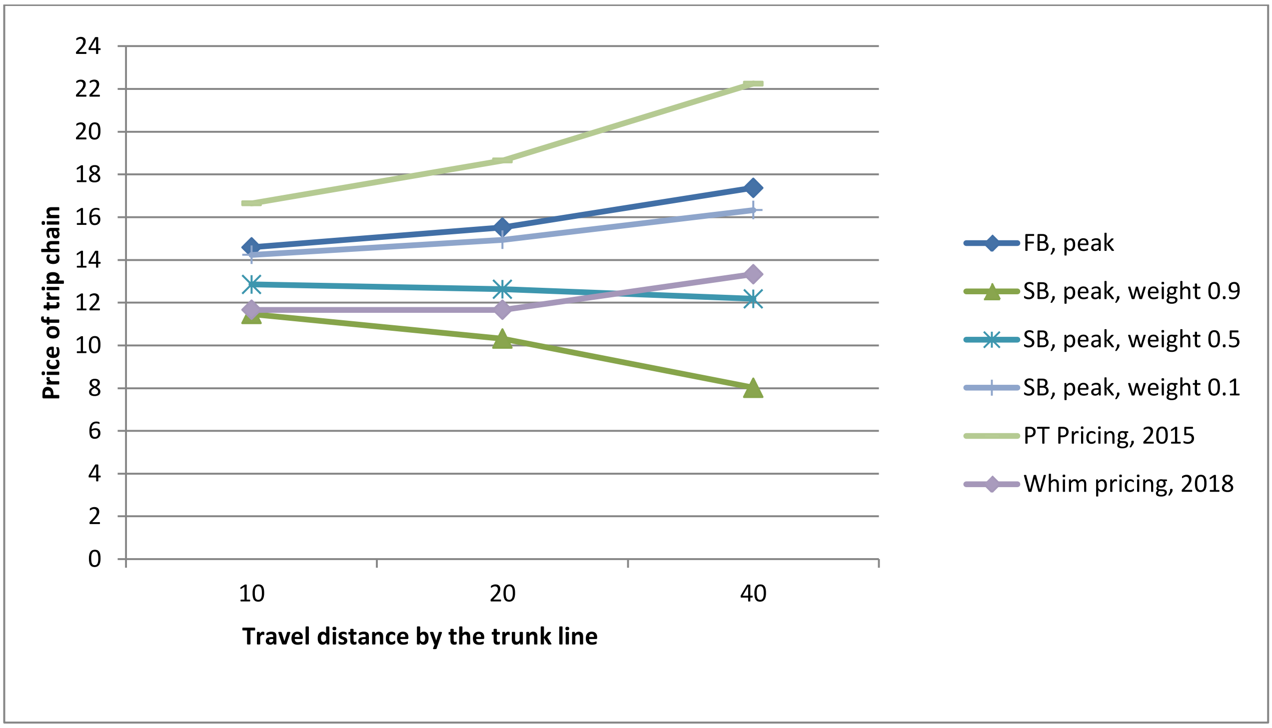

Figure 2. First best and second best prices of the multimodal trip chain (taxi feeder and trunk line) with various market conditions and pricing of public transport and The Whim service in Finland.

4 Conclusions

We have presented model for multimodal trip chain and derived the first best and second best pricing policies for trip chains in the model. The costs in the model were defined as per kilometer enabling analysis of distances in pricing policies. The analysis of distances in multimodal trip chains is crucial, if cost structures of transport modes are very different as between trains and taxis. Taxi is one of the most expensive transport modes (both in terms of production and external costs) and train is one of the most cost-efficient modes (at least with sufficient demand). The combination of these two modes can be also cost-efficient alternative for private car especially if the distance travelled by the train is relatively long as the presented numerical examples also illustrated. The presented examples show that under certain market conditions second best prices are lower and optimal subsidies respectively higher for trip chains with longer distances, which is depending on the substitution between private car and public transport. In general, the development of the seamless multimodal travel services for public transportation can bring the transport modes closer to the perfect substitutes as the service level of public transport increases.

The presented model can be extended for more than two transport modes. The other relevant directions for extending the model are the new flexible transport services, non-motorized transport modes and congestion interactions between the modes. Moreover, empirical analysis with accurate and detailed estimates of cross-effects and cost components is needed for applying the model in transport policies.

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